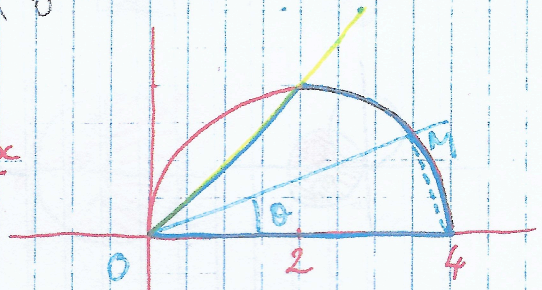
⑦ $\iint \sqrt{x^2+y^2}\,dx\,dy$

$D: \begin{cases} x^2+y^2-4x\le 0 \iff (x-2)^2+y^2\le 4 \\ y\le x \\ y\ge 0 \end{cases}$

$I=\int_{\theta=0}^{\pi/4}\int_{r=0}^{4\cos\theta} r\times r\,dr\,d\theta$

$\cos^3\theta = \dfrac{\cos(3x)+3\cos x}{4}$



$OM = 4\cos\theta$

$I=\int_{\theta=0}^{\pi/4}\frac{1}{3}(4\cos\theta)^3\,d\theta$

$I=\frac{16}{3}\int_{0}^{\pi/4}[\cos(3\theta)+3\cos(\theta)]\,d\theta$

$I=\frac{16}{3}\left[+\frac{1}{3}\sin(3\theta)+3\sin\theta\right]_0^{\pi/4} = \frac{16}{3}\left(+\frac{\sqrt2}{6}+\frac{3\sqrt2}{2}\right) = \frac{80\sqrt2}{9}$

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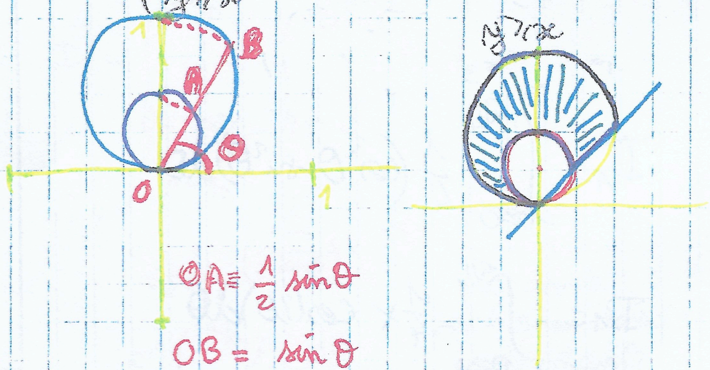
⑧ $\iint x\sqrt{x^2+y^2}\,dx\,dy$

$D \begin{cases} x^2+y^2-y\ge 0 \iff x^2+(y-\frac12)^2\ge\frac14 \\ x^2+y^2-2y\le 0 \iff x^2+(y-1)^2\le 1 \\ y\ge x \end{cases}$



$OA = \frac{1}{2}\sin\theta$

$OB = \sin\theta$

$I=\int_{\theta=\frac{\pi}{4}}^{\pi}\int_{r=\frac12\sin\theta}^{\sin\theta} r\cos\theta\times r\times r\,dr\,d\theta$

$=\int_{\theta=\frac{\pi}{4}}^{\pi}\cos\theta\left[\frac{r^4}{4}\right]_{\frac12\sin\theta}^{\sin\theta}d\theta$

$=\int_{\pi/4}^{\pi}\frac14\cos\theta\ \sin^4\theta\left(1-\frac{1}{16}\right)d\theta$

$=\frac{15}{64}\int_{\pi/4}^{\pi}\cos\theta.\sin^4\theta.\,d\theta$

$=\frac{15}{64}\left[\frac15\sin^5\theta\right]_{\pi/4}^{\pi}$

$= -\frac{3}{64}\times\left(\frac{\sqrt2}{2}\right)^5 = \frac{-3}{256\sqrt2}$